A new perspective on DGP gravity

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ABSTRACT: We examine brane induced gravity on codimension-1 branes, *a.k.a* DGP gravity, as a theory of five-dimensional gravity containing a certain class of four-dimensional branes. From this perspective, the model suffers from a number of pathologies which went unnoticed before. By generalizing the $5D$ geometry from Minkowski to Schwarzschild, we find that when the bulk mass is large enough, the brane hits a pressure singularity at finite radius. Further, on the self-accelerating branch, the five-dimensional energy is unbounded from below, implying that the self-accelerating backgrounds are unstable. Even in an empty Minkowski bulk, standard Euclidean techniques suggest that the spontaneous nucleation of self-accelerating branes is unsuppressed. If so, quantum effects will strongly modify any classical intuition about the theory. We also note that unless considered as $\mathbb{Z}_2$-orbifold boundaries, self-accelerating branes correspond to 'wormhole' configurations, which introduces the usual problematic issues associated with wormholes. Altogether these pathologies present a serious challenge that any proposed UV completion of the DGP model must overcome.

Contents

1. Introduction

The observed late-time acceleration of our universe [1] presents an enormous puzzle for theoretical physicists. There has been a great deal of effort invested in the search for a self-consistent modification of gravity as a way to address this problem, instead of introducing new matter contributions to Einstein gravity such as the cosmological constant. Perhaps the most notable examples of this approach are braneworld modifications [2–5] and theories with new scalars disguised as $f(R)$ terms [6]. The brane induced gravity model in $5D$ (henceforth refered to as DGP gravity, for short) [3] in particular has received special attention, mainly because it gave rise to the *self*-accelerating (SA) backgrounds. These solutions describe de Sitter cosmology without a nominal cosmological constant, or tension, on the brane, as have been written down in [7, 8]. They are similar in spirit to the original inflationary models of Starobinsky, found in higher derivative gravity without cosmological constant before the full power of inflationary dynamics was realized [9]. At first, the SA solutions appeared to evade any inconsistencies and in particular, the ghost problem of the GRS model [10, 11]. However, subsequent investigations demonstrated that a ghost appears on the SA brane [12, 13]. The appearance of both ghosts and tachyons was confirmed by the careful analysis of fluctuations around the SA solutions in [14, 15], which also showed that the normal branch of solutions remain free of ghosts and tachyons. While the appearance of ghosts and tachyons would seem to present a major problem for the DGP model, differing views have emerged as to the severity of this pathology [16, 17] — in particular, it is has been argued that strong coupling phenomena may somehow alter the

naive picture of instabilities. On the other hand, the exact shock wave analysis revealed some pathological singularities on the SA branch of solutions even beyond perturbation theory [18]. It is therefore clear that the issues concerning ghosts and tachyons remain a major obstacle to working with the DGP model as a reliable and practical description of our universe.

Most discussions of the DGP gravity to date focussed on the four-dimensional physics that brane observers might experience. Here we take a different point of view and examine the DGP model as a theory of five-dimensional Einstein gravity coupled to an unusual source: the four-dimensional DGP branes. In developing this perspective we uncover a number of new pathologies, which went unnoticed before.

The first of these are seen by generalizing the $5D$ geometry from Minkowski to Schwarzschild. If the bulk mass exceeds a certain critical value, the brane will run into a pressure singularity at finite radius. Further, it is straightforward to see that on the SA branch the five-dimensional energy is unbounded from below. Because an SA brane excises the space where the naked singularity would have been, the remaining spacetime of negative $5D$ mass is nonsingular, and we can keep all these solutions in the spectrum of the theory. This reservoir of negative energies can be accessed by smooth deformations of the SA vacua, by turning on the radion field which encodes the bulk ADM mass. While such deformations are not immediately accessible to the gravitational modes on the normal branch backgrounds alone, since there is no normalizable radion in that case, backgrounds which contain any number of SA branes will reintroduce this instability.

Going back to a Minkowski bulk, we can use the standard semiclassical techniques in Euclidean quantum gravity to demonstrate that the mixing between empty $5D$ space and SA branes appears to be completely unsuppressed. This result would lead us to conclude that as long as the theory contains SA brane solutions, $5D$ Minkowski space cannot be a good approximation of the (quantum) vacuum of the theory. Of course, these calculations in the Euclidean path integral are subject to certain ambiguities — as we will discuss. Finally, we note that unless the DGP branes are considered to be $\mathbb{Z}_2$-orbifold boundaries, the self-accelerating branes are actually 'wormhole' configurations. This will automatically introduce the usual set of problematic issues associated with such geometries [19].

Our results complement the perturbative exploration of pathologies in the DGP model from the viewpoint of brane-localized observers, which revealed ghosts and tachyons in the spectrum of small perturbations about the SA solutions [14, 15, 20, 21]. We uncover the present pathologies using the nonperturbative classical theory, which probes the full nonlinearities of the gravitational theory, in contrast to the perturbative analysis of [14, 15]. This shows that the nonlinearities, on their own, do not remedy the perturbative maladies of the model. The present calculations are extremely simple, utilizing the high degree of symmetry expected to characterize physically relevant backgrounds for cosmology at the Hubble scales. This simplifies the technical aspects of the work in contrast to the general perturbative considerations of [14, 15]. The results which we obtain provide clear and simple examples of how strong coupling effects *do not* cure sub-crossover dynamics on their own. Indeed, irrespective of any strong coupling phe-

nomena, the solutions with arbitrarily negative energies remain accessible to classical dynamics, at least in some particular dynamical channels. On the other hand, this also affirms that in its present form the low energy description better be very sensitive to higher-derivative, strong coupling corrections if one wants to maintain the belief that such phenomena may alter our conclusions. We stress that while the pathologies found here paint a disturbing picture of the DGP model, we cannot rule out the possibility that this model may have a sensible UV completion. It appears clear, however, that these pathologies do present a severe challenge for any such proposed UV completion. While the model at its current stage may motivate the search for a complete theory, given the severity of the pathologies revealed here as well as the presence of tachyonic and ghost-like perturbations, it seems premature to use its background solutions, and in particular the self-accelerating cosmologies, as a basis for constructing any detailed observational tests, such as those pursued in [22, 23] and many subsequent works. Moreover, our results seem to identify the self-accelerating branes as the source of instability. Thus this suggests that if stable UV completions of the DGP model should exist, they may not allow for self-accelerating branes, altogether invalidating the utility of such solutions. We note that the perspective and calculations developed here are applicable more generally in a variety of other settings and may be used as a diagnostic in other higher dimensional models or braneworld scenarios, of interest as a modified theory of gravity.

We should mention here a recent interesting work [24], where the authors sought configurations describing transitions from SA branes to N branch solutions, that look like normal branch bubbles on the SA brane. Such solutions require domain walls separating different de Sitter phases, and the authors of [24] argued that smooth walls supported by positive definite field theories may not exist. On this ground, these authors suggested that perhaps the perturbative ghost of DGP model might be less ominous than one may think. Now, while these arguments may be questioned, we should stress that our arguments are completely orthogonal to the investigation of [24], and show that nonperturbative instabilities *do exist*. Hence, at the level of the semiclassical theory, the ghost of DGP is not tamed, but comes out with a vengeance.

The paper is organized as follows: section 2 provides a brief overview of the DGP model. In section 3, we examine the various cosmological solutions of this model where the bulk spacetime corresponds to the five-dimensional Schwarzschild solution. For a large enough (positive) mass in the bulk, we show that the brane runs into a pressure singularity at finite radius. We further point out that on the SA branch, the negative-mass Schwarzschild subfamily still represent a class of smooth solutions of the DGP theory, yielding states which have a negative energy from the five-dimensional perspective. In section 4, we turn to semiclassical calculations describing the nucleation of a self-accelerating brane in empty $5D$ Minkowski space. We close with a discussion of our results in section 5. Appendix A examines the solutions of section 3 in the limit where the Schwarzschild mass parameter is small and connects this limit with the perturbative analysis of [14].

2. The DGP model

The $5D$ DGP model, as we study it here, is described by the following action:

$$S = 2M_5^3 \int_{\text{bulk}} \sqrt{-g}R + 4M_5^3 \int_{\text{brane}} \sqrt{-\gamma}K + \int_{\text{brane}} \sqrt{-\gamma}\left(M_4^2\mathcal{R} - \sigma + \mathcal{L}_{\text{matter}}\right) \quad (2.1)$$

where g_{ab} is the bulk metric with corresponding Ricci tensor R_{ab}. The brane has induced metric $\gamma_{\mu\nu}$ with corresponding Ricci tensor $\mathcal{R}_{\mu\nu}$. Its extrinsic curvature is given by $K_{\mu\nu} = -\frac{1}{2}\mathcal{L}_n\gamma_{\mu\nu}$, the Lie derivative of the induced metric, with respect to the unit normal n^a, pointing *into* the bulk. For the most part, we will work with the solutions which are $\mathbb{Z}_2$-symmetric about the brane. This may be thought as a simplifying constraint which focuses our attention on a special class of solutions. Alternatively, it may be that the brane is actually a $\mathbb{Z}_2$-orbifold in which case we would think of this as a boundary of the five-dimensional spacetime. On a pragmatic level, the $\mathbb{Z}_2$-symmetry means we only ever work with one side of the bulk in the following.

The key feature of the DGP model is the intrinsic curvature term appearing on the brane. In general, one might think that such a term should be induced on the brane by matter loop corrections [25–27] or finite width effects [28]. However, as we comment below, the phenomenologically interesting case requires a hierarchically much larger brane Planck scale M_4 than the bulk Planck scale M_5 [3], which has not been easy to realize naturally, and has been questioned on theoretical grounds [29]. In the brane action, we have explicitly extracted the brane tension σ out of the matter Lagrangian $\mathcal{L}_{\text{matter}}$. The tension term can be viewed as encoding the vacuum energy of the brane-localized fields. Then, the governing equations of motion in the bulk are the vacuum Einstein equations

$$G_{ab} = R_{ab} - \frac{1}{2}Rg_{ab} = 0. \quad (2.2)$$

The boundary conditions at the brane are simply the Israel junction conditions extended to the present case:

$$4M_5^3 K_{\mu\nu} + 2M_4^2\left(\mathcal{R}_{\mu\nu} - \frac{1}{6}\mathcal{R}\gamma_{\mu\nu}\right) - \frac{\sigma}{3}\gamma_{\mu\nu} = T_{\mu\nu} - \frac{1}{3}T\,\gamma_{\mu\nu} \quad (2.3)$$

where $T_{\mu\nu} \equiv -\frac{2}{\sqrt{-\gamma}}\frac{\partial(\sqrt{-\gamma}\mathcal{L}_{\text{matter}})}{\partial\gamma^{\mu\nu}}$. We include it here for generality, although it will vanish for the solutions which we will consider below.

There are a variety of different approaches to solving the coupled equations (2.2), (2.3). Here we will first solve the bulk equations and then use the Israel junction conditions to determine the trajectory of the brane in this bulk. For our purposes, the first step amounts to choosing a known solution of the five-dimensional vacuum Einstein equations. Of course, the simplest of these is just five-dimensional Minkowski space:

$$ds^2 = g_{ab}dX^a dX^b = -dt^2 + dr^2 + r^2 d\Omega_3^2. \quad (2.4)$$

The brane sweeps out a surface in this background at which two copies of the bulk geometry are stitched together (or an orbifold boundary condition is imposed). A simple but interesting brane geometry to consider is a homogeneous cosmology

$$ds^2 = \gamma_{\mu\nu}dx^\mu dx^\nu = -d\tau^2 + a(\tau)^2\,d\Omega_3^2, \quad (2.5)$$
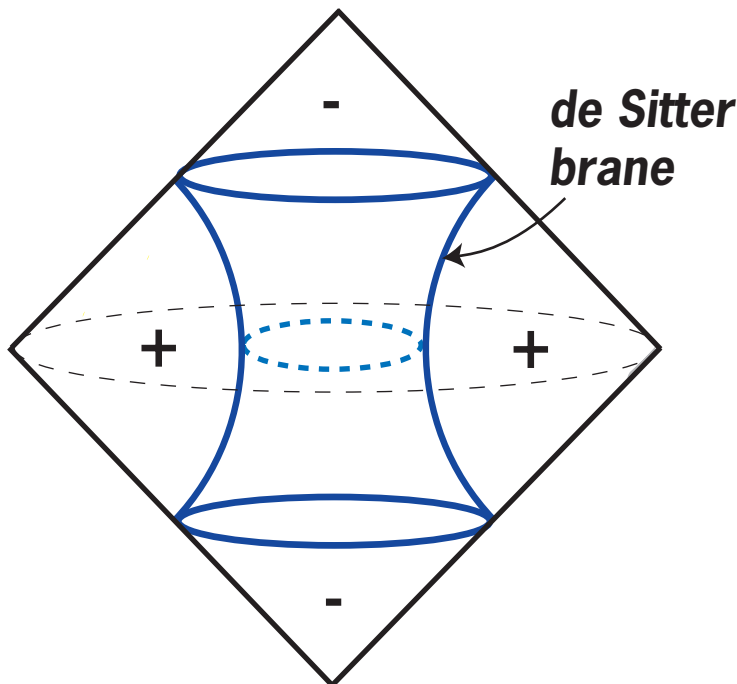
Figure 1: Embedding of a de Sitter brane in a flat 5D bulk. The brane world volume is the hyperboloid in the Minkowski bulk. The normal branch corresponds to keeping the interior of the hyperboloid, the self-accelerating branch the exterior.

where $a(\tau)$ specifies the proper size of the brane as a function of its proper time τ. To specify the brane's trajectory or embedding in the background (2.4), we set $r = a(\tau)$. Omitting the details of analysing the Israel conditions (2.3) for this case here[1] we just note that they yield the following interesting vacuum solutions on the brane [7, 8, 30]

$$a(\tau) = \frac{1}{H_\pm} \cosh(H_\pm \tau) \tag{2.6}$$

where

$$H_\pm \equiv \frac{1}{2} H_0 \left(\sqrt{1 + \frac{4\sigma^*}{H_0}} \pm 1 \right) . \tag{2.7}$$

Here we have introduced $\sigma^* = \sigma/12 M_5^3$ and $H_0 = 2 M_5^3/M_4^2$. These solutions describe de Sitter geometry on the brane with radius of curvature H^{-1}. As our construction elucidates, the brane can be viewed as a $4D$ hyperboloid of the same radius embedded in the $5D$ Minkowski bulk (see figure 1), generalizing the inflating domain walls of [31]. The choice of sign appearing in (2.7) arises because the construction left ambiguous which part of the bulk spactime was included. Indeed, in the minimal approach, we can treat the brane as a $\mathbb{Z}_2$ orbifold [3], in which case we identify the different sides of the bulk. Then the 'minus' sign denotes pasting together two copies of the region interior to the hyperboloid, and the 'plus' sign refers to pasting two copies of the exterior.

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[1] It corresponds to the special case $\mu = 0$ of the calculation in the next section.

The solution with H_- is commonly referred to as the *normal* branch whereas the solution with H_+ is referred to as the *self-accelerating* branch, a terminology which will become transparent shortly. The value of $H_0 = 2M_5^3/M_4^2$ in (2.7) is typically taken to be the current Hubble scale. This illustrates vividly the need for an exponential hierarchy between the Planck scales: given that M_4 is set to the observed four-dimensional Planck scale, $M_4 \sim 10^{19}\,\text{GeV}$, to get $H_0 \sim 10^{-33}\,\text{eV}$, one must take $M_5 \sim 10^{-3}\,\text{eV}$, some thirty orders of magnitude smaller than M_4. One must explain how to generate, and stabilize against radiative corrections, such a disparate ratio between the two Planck scales.

On the other hand, a new and very interesting feature of the theory is that even for vanishing tension, the self-accelerating solution gives rise to a de Sitter brane universe with $H = H_0$. The modification of gravity at large distances enables us to describe an accelerating universe in the absence of any vacuum energy whatsoever! In contrast, the normal branch gives rise to a flat Minkowski brane as $\sigma \to 0$, which may be less interesting for the phenomenology of an accelerating universe, but may still be a useful testing ground for other effects of gravity modified at large scales.

Returning to our geometrical picture of the branes as hyperboloids in Minkowski space, we note that the self-accelerating solution leads to a rather counter-intuitive result. Recall that in that case we keep the *exterior* of the hyperboloid. In the absence of the induced curvature of the brane, this would be consistent with a brane of *negative* tension. But this is not the case here. What has happened is that the induced curvature term enables us to mimic negative tension even when $\sigma > 0$. If we cast the brane equations (2.3) in the conventional form of the Israel junction conditions, we would have

$$4M_5^3 \left(K_{\mu\nu} - K\,\gamma_{\mu\nu} \right) = T_{\mu\nu}^{\text{eff}} \equiv -2M_4^2 \left(\mathcal{R}_{\mu\nu} - \frac{1}{2}\mathcal{R}\gamma_{\mu\nu} \right) - \sigma\,\gamma_{\mu\nu} + T_{\mu\nu} \qquad (2.8)$$

where the effective stress tensor for the brane comes from the variation of the total brane action, i.e., the third integral in eq. (2.1). Hence the right-hand side of the Israel conditions include the standard contributions coming from the brane tension and the brane matter, both of which will satisfy the usual positive energy conditions [32] (with positive σ). However, there is also a geometric contribution, i.e., the Einstein tensor of the intrinsic metric, which in general will not satisfy any positivity conditions. Note the overall minus sign in front of this term which arises here because we have put this geometric term on the matter side (*the wrong side*) of the equations in eq. (2.8). In particular, for a brane with a de Sitter geometry, as considered above, this term contributes $6\,M_4^2 H^2\,\gamma_{\mu\nu}$ to the effective stress tensor, i.e., this geometric contribution is equivalent to a negative brane tension $\sigma_{\text{geometry}} \equiv -6\,M_4^2 H^2$. This term comes to dominate on the self-accelerating branch, for which we may think in terms of the total effective tension being negative, $\sigma_{\text{eff}} = \sigma - 6\,M_4^2 H_+^2 = -12M_5^3 H_+$.

As commented above, the effective stress tensor will not in general satisfy any of the energy conditions typically considered in Einstein gravity [32] because of the geometric term appearing on the right-hand side of eq. (2.8). Hence one must worry that unusual or problematic results may arise from the brane and/or excitations of the brane. In particular, excitations of the brane geometry, e.g., four-dimensional gravitons, may contribute

negative energies from a five-dimensional perspective. To leading order, this contribution might be expected to vanish as the gravitons would satisfy (something-like) the four-dimensional Einstein equations and so their leading contribution to $T_{\mu\nu}^{\text{eff}}$ might then vanish. Of course, this question requires a detailed analysis to take into full account the couplings between fields on the brane and in the bulk. However, this is precisely the analysis provided in [14, 33] where it was found that certain perturbations about the self-accelerating solution are indeed ghost-like, as well as finding other tachyonic modes.

We close here with one final observation. If we relax the $\mathbb{Z}_2$ boundary conditions on the brane, then the conventional interpretation of the SA branch has the brane connecting two asymptotically flat regions of space. Hence this configuration provides what is widely known in the relativity literature as a 'wormhole'. In particular, while implicitly the two sides of the SA brane are taken to be disjoint Minkowski spaces, there is no reason a priori why these branes should not be connecting distant locations within the same five-dimensional space, which makes the brane's role as a 'wormhole' more evident. Hence we recall that, as has been extensively considered, there is a set of problematic issues inherent to such wormhole configurations, such as the appearance of closed time-like curves — see, for example, [19]. We do not consider these issues further here but the wormhole geometry will have interesting implications in the context of the semiclassical calculations in section 4. Of course, these problems are eliminated if the DGP branes are actually defined to be $\mathbb{Z}_2$-orbifolds. In such a case, we would think of the brane as a boundary of the five-dimensional spacetime and there would be no (independent) geometry on the other side.

3. Branes in a Schwarzschild bulk

3.1 Finding solutions

The cosmological solutions above were based on the simplest possible bulk, namely, $5D$ Minkowski space (2.4). It is straightforward to extend this procedure to other bulk solutions with spherical symmetry. In what follows, we apply this procedure to a more interesting bulk solution, a five-dimensional Schwarzschild black hole:

$$ ds^2 = g_{ab} dX^a dX^b = -f(r) dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_3^2 \,, \tag{3.1} $$

where

$$ f(r) = 1 - \frac{\mu}{r^2} \,. \tag{3.2} $$

Given the $5D$ action (2.1), the mass is given by [34]

$$ m = 12\pi^2 \, M_5^3 \, \mu \,. \tag{3.3} $$

In the context of DGP gravity, the possibility of using this bulk geometry was considered in passing in [7]. We will examine the corresponding solutions in full detail. A key observation will be that there is no obstacle to constructing smooth solutions on the SA branch when the mass (3.3) is arbitrarily negative, because the SA brane excises the naked singularity. We note that the authors of [12] considered in more detail a special subfamily of SA branes

in negative mass bulks. By demanding that the brane remains static they inferred a lower bound on the mass. However such a bound is artificial as it only implies that for a more negative bulk mass the brane with a fixed tension can't be static, but it crunches into a singularity, as we will discuss below.

Let us adopt the same ansatz for the brane geometry and trajectory as above

$$ds^2 = \gamma_{\mu\nu}dx^\mu dx^\nu = -d\tau^2 + a(\tau)^2 \, d\Omega_3^2 \,. \tag{3.4}$$

with $r = a(\tau)$. To evaluate the Israel junction conditions (2.3), we must first examine the embedding geometry more carefully. As well as the tangent vectors along the three-sphere, there is a (future-pointing) time-like tangent vector on the brane which may be expressed as[2]

$$u^a = \left(\frac{dt}{d\tau}, \frac{dr}{d\tau}, 0, 0, 0\right) = \left(\frac{1}{f(a)} \left(f(a) + \dot{a}^2\right)^{1/2}, \dot{a}, 0, 0, 0\right) \,, \tag{3.5}$$

normalized such that $u \cdot u = -1$. The normal to the brane is given by

$$n^a = \epsilon \left(\frac{\dot{a}}{f(a)}, \left(f(a) + \dot{a}^2\right)^{1/2}, 0, 0, 0\right) \,, \tag{3.6}$$

with $n \cdot n = +1$ and $u \cdot n = 0$. Here $\epsilon = \pm 1$, which corresponds to the ambiguity of which part of the bulk is included in the construction, noted above. With $\epsilon = -1 \, (+1)$, n^a points towards the black hole (asymptotic infinity) and we keep the interior (exterior) region.

Now we can evaluate each of the contributions in (2.3). One finds that the extrinsic curvature is

$$K_{ij} = -\frac{1}{2}\mathcal{L}_n\gamma_{ij} = -\frac{\epsilon}{a}\left(f(a) + \dot{a}^2\right)^{1/2} \gamma_{ij} \tag{3.7}$$

where (i, j) indicate directions on the S^3. With the brane geometry (3.4), one easily evaluates the intrinsic curvatures as

$$\mathcal{R}_{ij} - \frac{1}{6}\mathcal{R}\gamma_{ij} = \left(\frac{\dot{a}^2}{a^2} + \frac{1}{a^2}\right)\gamma_{ij} \,. \tag{3.8}$$

Evaluating (2.3) for the three-sphere directions yields an expression which we write as

$$-\epsilon \, H_0 \left(\frac{\dot{a}^2}{a^2} + \frac{f(a)}{a^2}\right)^{1/2} + \frac{\dot{a}^2}{a^2} + \frac{1}{a^2} - H_0\sigma^* = 0 \,. \tag{3.9}$$

To understand this result better, we first re-express it in a form reminiscent of the Friedmann equation

$$\frac{\dot{a}^2}{a^2} + \frac{1}{a^2} = \frac{H_0^2}{2} + H_0\sigma^* + \epsilon\frac{H_0^2}{2}\left(1 + \frac{4\sigma^*}{H_0} - \frac{4\mu}{H_0^2}\frac{1}{a^4}\right)^{1/2} \tag{3.10}$$

$$\underset{a\to\infty}{\simeq} \frac{H_0^2}{2} + H_0\sigma^* + \epsilon\frac{H_0^2}{2}\left(1 + \frac{4\sigma^*}{H_0}\right)^{1/2} - \frac{\epsilon\,\mu}{a^4}\left(1 + \frac{4\sigma^*}{H_0}\right)^{-1/2} + \ldots$$

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[2]Implicitly here and in the following, we assume that $f(a) > 0$, i.e., the brane is outside of the black hole horizon.

where we have used (3.2). In the second line, we have expanded the right-hand side for large a and we can interpret the result in terms of the standard four-dimensional Friedmann equation. The first contribution corresponds to that of a cosmological constant. We recognize this expression as H_+^2 for $\epsilon = +1$ and H_-^2 for $\epsilon = -1$. The next contribution with a $1/a^4$ dependence matches that of a radiation gas with a density proportional to $\epsilon \mu$ — an effective 'dark radiation' that is commonplace in braneworld cosmology [35]. Note that this 'holographic' effect has an unusual character for the SA brane. While a positive μ induces a positive energy density on the N brane, the effective energy density is negative on the SA brane [36]. That is, the holographic or four-dimensional interpretation might be that the SA brane supports some additional ghost-like matter in the $\mu > 0$ background. Of course, perhaps this is simply another example of the often noted result that the bulk contributions to the effective stress-energy on the brane [37 – 39] may be negative. A nonvanishing μ also induces an infinite series of higher order terms proportional to $1/a^{4n}$ — many of which would be expected to be negative.

Setting $\mu = 0$, we recover the simple case examined in the previous section and one can easily verify that eqs. (2.6) and (2.7) give a solution of (3.10). In general, we have no analytic solutions for (3.10). However to gain intuition for the solutions we can rewrite the equation as the Hamiltonian constraint of a classical point particle,

$$\dot{a}^2 + U(a, \epsilon, \mu) = -1 \,, \tag{3.11}$$

moving in an unusual potential:

$$U(a, \epsilon, \mu) = -\frac{H_0^2}{2} a^2 \left(1 + \frac{2\sigma^*}{H_0} + \epsilon \left(1 + \frac{4\sigma^*}{H_0} - \frac{4\mu}{H_0^2} \frac{1}{a^4} \right)^{1/2} \right) \,. \tag{3.12}$$
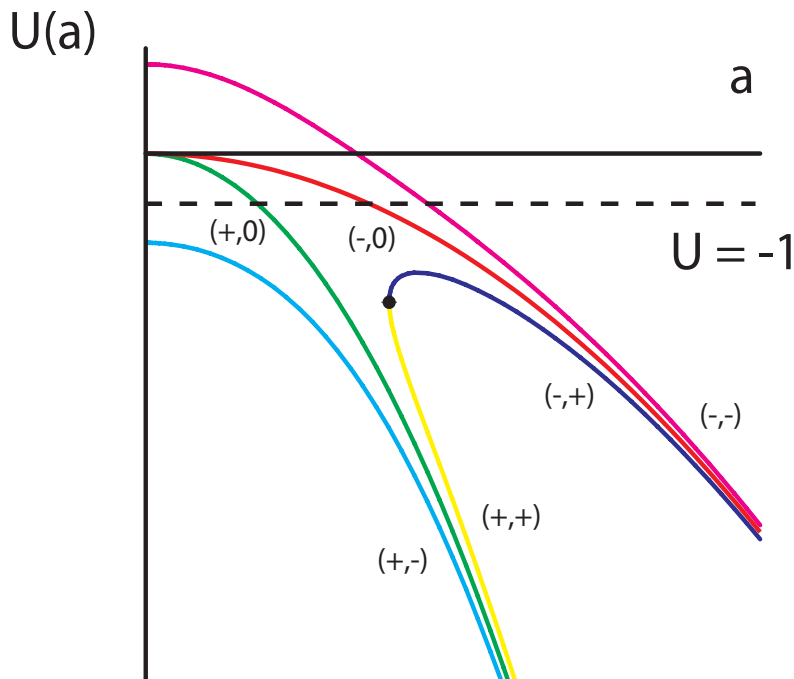
The effective potential $U(a, \epsilon, \mu)$ is plotted in figure 2 for a variety of parameter choices. Let us first reconsider the special case $\mu = 0$ from this perspective. In this case, the potential reduces to a simple inverted harmonic potential,

$$U(a, \epsilon, \mu = 0) = -H_\pm^2 a^2 \tag{3.13}$$

where H_- (H_+) corresponds to the choice $\epsilon = -1$ ($+1$). Given that the effective energy is a fixed negative quantity, the trajectories of interest approach the origin from infinity, bounce off the potential at $a_{\min} = 1/H_\pm$, defined in figure 2 as the point of intersection of $U(a)$ with the line $U = -1$, and then head back to infinity. Of course, this is precisely the behaviour shown by the analytic solutions (2.7), describing de Sitter spaces on the brane in global coordinates.

An alternative visualization of the DGP cosmology is given by plotting the solution as a trajectory in phase space. Caution must be exercised as the phase plot is *not* that of an autonomous dynamical system, and hence can display irregularities, nonetheless some qualitative features are very easily extracted. Explicitly, to get a dimensionless plot, rescale variables:

$$\hat{t} = H_0 t \,, \quad \hat{a} = H_0 a \,, \quad \hat{\mu} = H_0^2 \mu \,, \quad \hat{\sigma} = \sigma^*/H_0 \tag{3.14}$$

Figure 2: The effective potential $U(a, \epsilon, \mu)$ describing the brane trajectories in various cases, classified by the signs of ϵ, μ. The black dot indicates the singular radius a_{crit} which demarcates the $\epsilon = \pm 1$ solutions with positive bulk mass, and may be accessible by evolution depending on the precise position of the line $U = -1$, which can shift up and down as a function of the parameters.

and define the phase coordinates as

$$X = \frac{1}{\hat{a}} \qquad Y = \frac{\dot{\hat{a}}}{\hat{a}} \; . \tag{3.15}$$

The Friedmann equation (and hence the trajectory in phase space) coming from the square of (3.9) now becomes

$$(X^2 + Y^2)^2 - (1 + 2\hat{\sigma})(X^2 + Y^2) + \hat{\sigma}^2 + \hat{\mu}X^4 = 0 \tag{3.16}$$

A set of typical plots (for the SA branch) is shown in figure 3. It is easy to see that in the absence of a bulk black hole, the trajectory is a circle in the (X, Y) plane, whose radius is fixed by the brane tension. We also see that for very small black hole masses, the cosmology is very slightly perturbed, with the SA brane being *repelled* by positive mass black holes. Finally, we also notice that for large (positive) black hole masses, the trajectories exhibit pathologies, which will be described in section 3.2.

It is interesting to note that if $\mu \to 0$, we can regard our $\mu \neq 0$ solutions as a small perturbation of the $\mu = 0$ case. Hence in this limit, one should be able to relate our new solutions to the fluctuation analysis [14, 33]. We leave the details of this analysis to appendix A but observe that one finds that the perturbation introduced by small μ is related to the homogeneous mode of the 'radion'. In particular, on the SA branch, this radion mode is normalizable and so describes a dynamical field in the theory. Further,
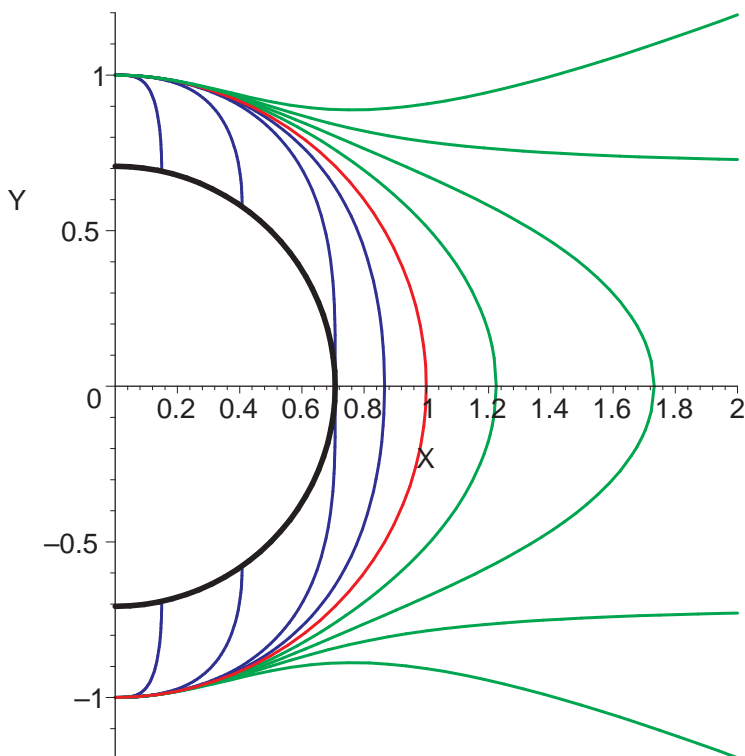
Figure 3: A sample of phase plane trajectories for the DGP cosmology with positive and negative bulk black holes masses for the SA branch solution. We have taken $\sigma = 0$ for this plot. The red circle (of radius one) represents the $\hat{\mu} = 0$ or standard DGP cosmology. The green plots are with negative bulk black hole masses ($\hat{\mu} = -1/3, -2/3, -1, -3/2$), and the blue plots correspond to positive bulk black hole masses ($\hat{\mu} = 1/3, 1, 9, 500$). Note that for $\hat{\mu} \geq 1$, the trajectories terminate on the pressure singularity (described in the text) which corresponds to the solid black circle at $X^2 + Y^2 = 1/2$. Similarly trajectories with $\hat{\mu} < -1$ asymptote to $(X, Y) \rightarrow (+\infty, \pm\infty)$, corresponding to reaching the singularity at $a = 0$.

on the SA brane, since the radion is always either a ghost or a tachyon, it is expected to be associated with an instability. Indeed, in the appendix we will show that at distances much greater than the gravitational radius μ of the bulk mass m, where we can trust linearized gravitational fields in the bulk, the bulk mass parameter is completely encoded by the nontrivial radion configuration. The mass is zero if the radion vanishes, and its sign is determined by the sign of the radion. Now, on the N branch, the radion is not a dynamical field in the theory, as it is not normalizable. Of course, this reflects the fact that our full solutions on the N branch contain a black hole (or a naked singularity) at the center of the bulk space. However, describing the latter requires the full nonlinear gravity theory no matter how small μ is and so goes beyond a linearized analysis presented in [14, 33]. Thus on the normal branch μ, and therefore the bulk mass m, can be regarded as boundary conditions from the point of view of the linearized theory. This does not exclude the possibility that μ may be changed by processes whose initial stages may be reliably described within the perturbative approach of the linearized analysis, but the complete evolution necessarily goes into a nonlinear regime. For example, one might consider a

uniform spherical wave emitted from the brane with a small amplitude and then collapses at the center of the bulk space to form a black hole.

While the above comments are largely observations about mathematics, we want to comment that the physics is distinguished here between the SA and N branches. If we consider generic local processes which result in transmitting energy off the brane into the bulk, for the closed cosmology of the N branch, this energy simply disperses within the finite spatial slices of the bulk geometry. In contrast, on the SA branch, the analogous processes will generically send out some energy which 'leaks out' to infinity and so inevitably μ decreases. Thus, on the SA branch, and in general in the presence of SA branes, even if we start with a bulk geometry where $\mu = 0$, we will be able to access $\mu \neq 0$, and in particular $\mu < 0$. Further, given the discussion above, we should be able to describe this evolution within the linearized theory as radion dynamics. In the bulk we can always go sufficiently far from the SA brane that the linearized description will be valid, and so it is hard to see how such processes can be avoided without a complete exclusion of SA branes. On the other hand, if we ban naked singularities, such processes on N-branch will not create configurations with $\mu < 0$.

3.2 Positive black hole mass in the bulk and pressure singularities

We now consider the effect of a *positive* bulk mass ($\mu > 0$). As we have just discussed, this could be generated perturbatively by the radion on the SA branch, or through a nonperturbative altering of the boundary conditions on the N branch. Typical potentials (3.12) for this case are shown in figure 2. For any value of μ, there are of course *two* branches for the potential, one corresponding to the N branch ($\epsilon = -1$), and one corresponding to the SA branch ($\epsilon = +1$). When μ is positive, the two branches connect to one another at a singular point, indicated with the black dot.

Let us begin by looking at the SA branch ($\epsilon = +1$). At this point it useful to consider the phase plane plot given by figure 3. Here we are interested in positive bulk mass, so we should turn our attention to the blue trajectories. These clearly split into two families: connected trajectories and disconnected trajectories. The connected trajectories correspond to small values of μ, and pass safely through the Y-axis. These trajectories are qualitatively similar to the $\mu = 0$ trajectory. A glance at the $(+, 0)$ curve for the potential (3.12) in figure 2 indicates the following generic behaviour: the brane falls in from large a until it hits a bounce at some finite value a_b and then expands back out again. The trajectory is completely non-singular and the bounce occurs when the potential crosses $U = -1$. It follows that

$$a_b = \frac{1}{\sigma_*} \left[\frac{1}{2} + \frac{\sigma_*}{H_0} - \sqrt{\frac{1}{4} + \frac{\sigma_*}{H_0} - \sigma_*^2 \mu} \right]^{1/2} \tag{3.17}$$

Note that the brane always stays outside of the black hole horizon since $a_b \geq \mu^{1/2}$. It is easy to see that a_b increases as we increase μ, which agrees with our earlier statement that the SA brane is *repelled* away from positive mass black holes. The repulsive effect of $\mu > 0$ on the SA branch is expected given the analysis of the effective Friedmann equation (3.10) in which a negative density radiation term appears with positive μ and $\epsilon = +1$. However,

since we are keeping the exterior or asymptotic region of the black hole geometry (3.1) and, at the same time, the brane has a negative effective tension, we cannot gain any futher intuition by considering the global geometry.

The disconnected trajectories occur for larger values of μ, greater than some critical value μ_0, to be determined shortly. These trajectories terminate on the bold black circle in figure 3, at which point the brane runs into/out of a pressure singularity. This point also occurs precisely on the bold black dot on the $(\pm, +)$ curve in figure 2. So what exactly is going on? Suppose the brane is falling in from a large a. At some point it will reach the black dot, at a critical value given by

$$a_c = \left[\frac{\mu}{H_0^2/4 + H_0\sigma^*} \right]^{1/4} . \tag{3.18}$$

For $a < a_c$, the potential (3.12) is complex, so this regime is unphysical. As $a \to a_c$ from above, the potential terminates with an infinite slope at the black dot. At this point, $\dot{a}$ remains finite, but $\ddot{a}$ (and any higher derivatives) diverge.

We can confirm that this is a real physical singularity on the brane by calculating the Ricci curvature for the induced metric, e.g.,

$$\mathcal{R} = 6 \left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a} \right)^2 + \frac{1}{a^2} \right) , \tag{3.19}$$

Clearly, this expression blows up at $a = a_c$, since $\ddot{a}$ diverges as $a \to a_c$ and so we have a genuine singularity on the brane. This occurs at some finite radius which is otherwise ordinary from the perspective of the bulk geometry. For a phenomenologically interesting range of parameters (e.g., the Schwarschild radius of the black hole is smaller than the Hubble scale on the brane), a_c is typically far outside of the bulk black hole event horizon, as can be seen by comparing it with the bulk black hole horizon radius $a_h = \mu^{1/2}$:

$$\frac{a_h^4}{a_c^4} = \mu \left(H_0^2/4 + H_0\sigma^* \right) = \frac{1}{12\pi^2} \frac{m}{M_5} \left(\frac{M_5^4}{M_4^4} + \frac{\sigma}{6M_5^2 M_4^2} \right) . \tag{3.20}$$

This singular behaviour observed above resembles certain 'sudden' cosmological singularities studied recently [40], induced by the diverging pressures on certain spacelike hypersurfaces as opposed to diverging energy densities. In fact, these singularities are really reminiscent of the well known pressure divergences in the McVittie solution [41], which describes the field of a spherically symmetric mass in an FRW universe. Here, we have the field of a black hole, and the matter exterior to it is given by a thin brane. In the McVittie case also, the singularity of the solution induced by the diverging pressure is spacelike, and in the past extends well beyond the region where the black hole horizon of the fixed mass would have been.

So, on the SA branch, when the black hole mass is small and positive ($0 \leq \mu < \mu_0$), the brane follows a non-singular trajectory, falling in from large a, reaching a minimum at the bounce a_b, and expanding back out again. Here there is no pressure singularity because for these values of μ the brane hits the bounce before it can reach the point where

the singularity would occur. In other words, $a_b > a_c$. For larger values of the black hole mass ($\mu > \mu_0$), the situation is reversed: a brane falls in from large a and hits a pressure singularity at a_c before it gets the chance to bounce. This corresponds to the case $a_b < a_c$. Clearly we can calculate the critical value,[3] μ_0 by setting $a_b = a_c$, and solving for μ,

$$\mu_0 = \frac{1}{H_0} \frac{(\sigma^* + H_0/4)}{(\sigma^* + H_0/2)^2} . \tag{3.21}$$

The behaviour on the N branch ($\epsilon = -1$) is fairly similar. Without going into details, we note that for small enough $\mu > 0$, the behaviour is qualitatively the same as for $\mu = 0$. Glancing at the curve $(-, 0)$ for the relevant potential in figure 2, we see that the brane follows a non-singular trajectory, falling into some minimum radius and bouncing out again. This time, the position of the bounce *decreases* as μ increases. This means that a N brane is *attracted* towards a positive mass black hole. Of course, this is exactly in accord with our intuition that there is now a black hole at the center of the bulk geometry which exerts a gravitational attraction on the brane. Similarly, the appearance of an effective radiation term (with a positive density) in the effective Friedmann equation (3.10) also reflects the attractive effect of positive μ with $\epsilon = -1$.

At large enough values of μ, the brane will once again run into a pressure singularity. This corresponds to following the curve $(-, +)$ in figure 2, hitting the singularity at the black dot. For completeness, we note that on the N branch, there is in fact a third scenario. This occurs for intermediate values of μ, and corresponds to the case where the $(-, +)$ curve in figure 2 crosses the line $U = -1$, but the black dot appears below the line. Then there are two possibilities: either we have a bouncing trajectory with no singularity (as for small μ), or we have a brane that flows out of the pressure singularity, expands out to a maximum radius, and then contracts until it hits another pressure singularity. In addition, tuning μ such that $U_{\max} = -1$ also allows for special trajectories where the brane is static and sits at a fixed radius corresponding to the maximum.

Although the pressure singularity occurs for large enough μ on *both* branches, the effects are actually very different, depending on which branch (N or SA) you are on. To see this, let us examine the nature of this pressure singularity more closely. Intuitively, we expect that this singularity will be very dangerous because the pressure divergence will yield to the perturbations of the fluid dominating the universe becoming superluminal. Indeed, the speed of sound of such perturbations is given by $c_S^2 = \frac{\partial p}{\partial \rho}$, which will behave as $c_S^2 \sim p/\rho$ as $p \to \infty$. To check this, we can use a relativistic field theory as a probe, and ask what happens to its fluctuations as the universe approaches the singularity. Here we use the formalism for studying small perturbations in closed FRW universes that has been developed for inflation [42]. As in a simple recent application [43], we start with $\nabla^2 \chi = 0$, expand the field χ in spherical harmonics Y_{nlm} on S^3 as $\chi = \sum_{nlm} \chi(\eta)_n Y_{nlm}$, obeying $\vec{\nabla}^2 Y_{nlm} = -(n^2 - 1)Y_{nlm}$ for $n - 1 \geq l \geq |m|$ where $\vec{\nabla}^2$ is the Laplacian on S^3, transform to the conformal time coordinate $d\eta = d\tau/a$, and define the new field variable $\xi_n = a\,\chi_n$. A

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[3]The reader may note that equation (3.17) shows another critical value $\tilde{\mu}_0 = (\sigma_* + H_0/4)/(H_0\sigma_*^2)$. However, $\mu_0 < \tilde{\mu}_0$, and so the latter is not physically significant.

straightforward calculation yields the equation governing the time evolution of a particular radial quantum number n,

$$\xi_n'' + \left(n^2 - 1 - \frac{a''}{a} \right) \xi_n = 0 \,, \tag{3.22}$$

where the primes denote derivatives with respect to the conformal time η. Now, it is easy to see that $\frac{a''}{a} = a^2(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2})$ in terms of the original time τ. Further, differentiating equation (3.11) with respect to τ, we find $\ddot{a} = -\frac{1}{2}\frac{\partial U}{\partial a}$.

Now let us focus on what happens on the N branch. Consider the $(-,+)$ curve for the potential $U(a)$ in figure 2. As we approach the singularity (represented by the black dot), it is clear that the slope of potential is large and positive. It follows that as $a \to a_c$, $\frac{a''}{a} \to -\infty$, which converts equation (3.22) into an harmonic oscillator with a huge mass, for any value of the comoving momenta $\propto n$. Therefore the singularity has the effect of essentially decoupling the local dynamics of the field theory inhabiting the universe.

In contrast, on the SA branch, we can see from looking at the $(+,+)$ curve in figure 2 that the slope of the potential becomes large and *negative* as we approach the singularity. It follows that as $a \to a_c$, $\frac{a''}{a} \to +\infty$, indicating that a tremendous, exponential instability sets in on all scales. Clearly, this applies to the physical mode $\chi_n = \xi_n/a$ as well, because a remains finite throughout. Of course, this instability could lead to singularities when small perturbations are introduced to the (homogenous) SA solutions on the bouncing trajectories where a_b is not much bigger than a_c, since $\frac{a''}{a}$ still becomes very large near the bounce.

3.3 Negative mass in the bulk

We now turn our attention to the case of a negative bulk mass ($\mu < 0$). Of course, typically we would not consider the bulk geometry (3.1) with $\mu < 0$ because it contains a naked singularity at $r = 0$. However, in our construction of the SA branch solutions, we have excised the bulk geometry inside the brane world-volume and have kept only the exterior. Hence this construction removes the singularity and produces a (potentially) smooth solution for $\mu < 0$ and $\epsilon = +1$. Furthermore, such a solution can be generated perturbatively by exciting a normalizable radion on the $\mu = 0$ solution.

We can learn more about the types of SA solution that occur for negative bulk mass by considering the green trajectories in the phase plane plot (figure 3). Again they are split into two families: connected trajectories for small $|\mu|$, and disconnected trajectories for large $|\mu|$. The connected trajectories correspond to the bounces, where the brane falls into some minimum radius and bounces back out again. As $|\mu|$ increases, the minimum radius actually *decreases*, so, as expected, we conclude that the negative mass has an *attractive* effect on the SA brane. The disconnected trajectories correspond to $|\mu| > 1/H_0^2$, when we no longer have bounce solutions. The corresponding potential is given by the $(+,-)$ curve in figure 2. We can clearly see that now the brane approaches from infinity and 'crashes' into the singularity in the bulk at $a = 0$ (or the time reverse). Since a vanishes at this point (while $\dot{a}$ and $\ddot{a}$ remain finite), a singularity also appears from the brane perspective, e.g., the Ricci curvature for the induced metric 3.19 diverges. Unfortunately as it stands, the DGP model is inadequate to determine the subsequent evolution of the system. Some

obvious, albeit naive possibilities are 1) that the trajectory simply terminates at $a = 0$ and a naked singularity emerges in the bulk spacetime, or 2) that the trajectory might continue on to negative values a, which could be interpreted as the brane passing through itself (and the singularity) at the origin and emerging as an expanding brane with the opposite orientation. It is noteworthy that at 'early' times when the brane is still at finite radius, the solution is smooth and so defines sensible initial data.[4] Hence even if the full evolution seems to be singular, there is no obvious reason why these solutions should be ruled out as physically unacceptable.

In contrast, on the N branch ($\epsilon = -1$) if we take $\mu < 0$, the N branch construction will leave a naked singularity at the center of the bulk space, which makes the solutions much less appealing. Nevertheless, one could easily imagine that we are shielded from this singularity by the presence of another SA brane in this geometry, evolving concentrically with the N brane. The latter would, of course, not effect the dynamics of the N brane, which will always correspond to a bounce. This can be seen by considering the $(-, -)$ curve in figure 2. As $|\mu|$ increases, so does the minimum radius, and so the effect of a negative mass is repulsive on the N branch, as one might have anticipated.

All of this brings us to our main conclusion: with the SA branes, the DGP model admits physically sensible solutions for which the five-dimensional energy can be arbitrarily negative. That is, regarded as a five-dimensional theory of gravity and branes, the spectrum of the DGP model is unbounded from below. Having identified new negative energy configurations, it is natural to think that these will be excited in both classical and quantum processes. While finding explicit solutions which demonstrate the appearance of these negative energy states in various dynamical processes is difficult, it remains reasonable to assume that the theory should be unstable.

4. Bubbling SA branes

In considering possible instabilities and their description beyond perturbation theory, given that the SA branes appear as the source of the ills in the theory, it is natural to look at tunneling processes in which the SA branes might be spontaneously created in the Minkowski vacuum. In what follows we present a semiclassical calculation for such a process using standard techniques of Euclidean quantum gravity. With a straightforward application of these techniques, we find that spontaneous nucleation of SA branes in the bulk is unsuppressed. However, as we discuss below, these calculations are not without certain ambiguities. We should also note that the processes considered here are complementary to the possible instabilities revealed in the previous section since the calculations here are restricted to the case $\mu = 0$. It would be interesting to consider the interplay of these effects, which we leave for another day.

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[4]Beginning from this smooth initial data, the brane collapses to $a = 0$ and the resulting singularity can be seen by asymptotic observers. Hence cosmic censorship is violated in DGP gravity. But perhaps this is not so surprising given that the DGP branes do not satisfy any of the usual positive energy conditions [32], as observed above.

We will now apply the standard instanton techniques for the description of tunneling processes with Euclidean path integrals. The tunneling probability from one configuration into another in an existing Lorentzian geometry to leading order is

$$\mathcal{P} \propto e^{-\Delta S/\hbar}, \tag{4.1}$$

where the instanton action ΔS is the difference of the Euclidean bounce and background actions,

$$\Delta S = S_{\text{bounce}} - S_{\text{background}}. \tag{4.2}$$

In analogy with particle mechanics, we refer to the Euclidean solution describing the 'bubble of SA brane' at the center of five-dimensional flat space as the 'bounce'. The Euclidean action is given by

$$S_E = -2M_5^3 \int_{\text{bulk}} \sqrt{g}R - 4M_5^3 \int_{\text{brane} + \infty} \sqrt{\gamma}K - \int_{\text{brane}} \sqrt{\gamma}\left(M_4^2\mathcal{R} - \sigma\right), \tag{4.3}$$

where in the second integral, we have indicated the integration of the extrinsic curvature over asymptotic infinity. This is, of course, the standard Gibbons-Hawking term and in the present case the net effect of the background subtraction will be to cancel this term (which is otherwise divergent).

Upon Wick rotating to Euclidean signature, $t \to it_E$, the bulk space becomes simply $\mathbb{R}^5$

$$ds^2 = dt_E^2 + dr^2 + r^2 d\Omega_3^2, \tag{4.4}$$

with a ball of the radius H_+ removed and its surface identified with the Euclidean geometry of the brane. Indeed, on the brane, $\tau \to i\tau_E$ converts the de Sitter geometry of the SA branch to that of a four-sphere with

$$ds^2 = d\tau_E^2 + \frac{1}{H_+^2}\cos(H_+\tau_E)^2 \, d\Omega_3^2. \tag{4.5}$$

The instanton describing tunneling from empty space to a space containing a single SA brane will consist of one half of both of these geometries, but in squaring the wavefunction, the tunnelling probability (4.1) requires the action for the full Euclidean bounce. Note, that this construction is perfectly consistent with the $\mathbb{Z}_2$ orbifold condition. We can take two $5D$ Euclidean spaces with a ball removed, and the remaining spaces identified across the spherical boundary, and then analytically continue to Lorentzian signature at the equator, to get the SA brane. This will provide an 'interior' picture of the process, without any reference to the embedding space. Clearly, we can also consider the processes where $\mathbb{Z}_2$ condition is lifted.

Now, to calculate the action for the bounce, we first use the Euclidean version of the equations of motion (2.2), (2.3) to deduce that

$$R = 0, \qquad 4M_5^3 K + \frac{2}{3}M_4^2\mathcal{R} - \frac{4}{3}\sigma = 0, \tag{4.6}$$

where, as above, we have set all the matter contributions on the brane, except the tension, to zero. Hence the Euclidean action (4.3) for the instanton reduces to

$$S_{\text{bounce}} = -\frac{1}{3} \int_{\text{brane}} \sqrt{\gamma} \left(M_4^2 \mathcal{R} + \sigma \right) - 4 M_5^3 \int_\infty \sqrt{\gamma} K \,. \tag{4.7}$$

Plugging in the spherical geometry (4.5) above yields

$$S_{\text{bounce}} = -\frac{1}{3 H_+^3} \int d\Omega_3 \int_{-\frac{\pi}{2H_+}}^{\frac{\pi}{2H_+}} d\tau_E \, \cos(H_+ \tau_E)^3 \left(12 M_4^2 H_+^2 + \sigma \right) - 4 M_5^3 \int_\infty \sqrt{\gamma} K$$

$$= -\frac{8\pi^2}{9 H_+^4} \left(12 M_4^2 H_+^2 + \sigma \right) - 4 M_5^3 \int_\infty \sqrt{\gamma} K \,. \tag{4.8}$$

Now we must also consider the background contribution to the instanton action (4.2). The background spacetime is simply flat space and so the only contribution to the action is the asymptotic integral of the extrinsic curvature, precisely cancelling the second term in (4.8). Thus the final result for (4.2) is

$$\Delta S = -\frac{8\pi^2}{9 H_+^4} \left(12 M_4^2 H_+^2 + \sigma \right) = -\frac{8\pi^2}{3 H_+^4} \left(8 M_5^3 H_+ + \sigma \right) \,. \tag{4.9}$$

We immediately note that that the sign of either of these expressions is obviously *negative* for positive tension, $\sigma > 0$. Hence this action provides no suppression for the tunneling probability (4.1)!

Our interpretation of this result is certainly not that the tunneling probability is greater than one, but rather that the saddle-point approximation implicit in (4.1) is not a good one. A more careful analysis would be required to properly evaluate the tunneling probability. However, what is clear is that the breakdown of the saddle-point approximation implies that the tunneling is not strongly suppressed and so there is large mixing between the empty five-dimensional spacetime and that containing a SA brane. By extension, one can infer that there is a large mixing between all of the five-dimensional spaces containing any number of SA branes. Hence we are led to conclude that empty five-dimensional Minkowski space does not provide a good description of the quantum vacuum of the five-dimensional theory. Note that while this conclusion is in agreement with that found in the previous section, the present calculations are restricted to the zero-energy sector and make no reference to the negative energy states found previously.

Of course, it must be said that this rather dramatic conclusion hinges crucially on the sign of the Euclidean action in (4.9). At the same time, there has been a long-standing debate in the quantum cosmology literature [46 – 48] centered on precisely this sign. While we have nothing new to add to that debate, we would point out that there are significant differences between the setting of quantum cosmology[5] and the present description of nucleating SA branes. Interpretational ambiguities are naturally present in quantum cosmology where one considers the quantum birth of a *whole* closed universe from 'nothing'. In contrast, in

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[5]The sign ambiguity reappears in the present context when considering the N branch, in which the solutions correspond to closed five-dimensional universes. If one evaluates the Euclidean action (4.3) for a

the setting describing the nucleation of SA branes, we have a standard tunneling process describing the formation of a 'defect' in a larger empty spacetime, which serves as a *fixed* background. Hence we can take it as the usual background describing a metastable state, in which a tunneling process starts, and where the positivity of energy is defined by the ghost-free N-branch configurations. In this way we do not face the same interpretational issues as in the case of 'tunneling from nothing'. Although such interpretational issues are absent, this still does not guarantee the sign of the action in the tunneling amplitude. Indeed in a similar calculation, ref. [44] advocated the opposite sign to that chosen in (4.9) — however, we note that the discussion there overlooked the boundary contribution of the Gibbons-Hawking term, as well as focussing on hypothetical brane embeddings with $K_{\mu\nu} = 0$.

Let us consider then various choices for an unconventional analytic continuation in the path integral. The first and simplest choice would be choosing the opposite sign for the entire action, i.e., yielding $\mathcal{P} \propto e^{-|\Delta S|}$. However, in this case, we are implicitly applying the same unconventional continuation both to the saddle-point with the spherical Euclidean brane and to that for empty flat space. The latter seems rather problematic and so it is doubtful that this can be the correct choice for the continuation. As a compromise then, we could simply choose the conventional continuation for empty space and choose the opposite sign for the nontrivial saddle-point. However, in this case, the boundary terms will not cancel between the two solutions and so resulting ΔS is divergent. Hence this approach is also problematic. The final alternative which we consider here is that the unconventional continuation would only be applied to the brane action, i.e., we would only reverse the sign of the brane contributions in (4.3). This choice may seem natural since the SA branes have already been identified as the source of problems in DGP gravity and further this continuation leads to a finite positive ΔS. However, in this case, the analysis above must be reconsidered. In particular, the sign of the brane terms in the Euclidean equations of motion are also reversed and so the tunneling instanton will contain a Euclidean brane of radius $1/H_-$, rather than $1/H_+$ as above – recall $H_\pm$ are defined in (2.7). Hence, this approach does not seem to produce a consistent description of the tunneling event, in which the Euclidean brane naturally connects to the SA brane in Lorentzian signature. Hence, while we can not at present guarantee that the tunneling calculation does not require an unconventional analytic continuation, we would say that straightforward calculation of the Euclidean action seems the most appropriate choice.

It would be interesting to address this question using Hamiltonian techniques, such as in [49]. In a problem with some elements in common with the present case, this approach

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hemispherical brane, the Hartle-Hawking wavefunction for this closed universe becomes $\exp[-S_{HH}]$ with

$$S_{HH} = -\frac{4\pi^2}{9H_-^4}\left(12M_4^2 H_-^2 + \sigma\right) = -\frac{4\pi^2}{3H_-^4}\left(\sigma - 8M_5^3 H_-\right). \tag{4.10}$$

Note that the sign is again negative, as is standard for the Hartle-Hawking wavefunction [47], but alternate approaches would flip this sign [46, 48]. Note that taking the limit $M_5^3 \to 0$ in the second expression above, reproduces the standard result for four-dimensional Einstein gravity coupled to a cosmological constant Λ: $S_{HH} = -2\pi^2\Lambda/3H^4$ where $H = \sigma/6M_4^2$ and $\Lambda = \sigma/2$.

was recently used to argue the — perhaps — surprising result that the nucleation of domain walls comprised of ghost matter should be suppressed [50]. But we would also note that at least in certain situations, applying such Hamiltonian techniques to quantum gravity is known to produce erroneous results [51].

We must comment, however, that the conventional analytic continuation for the tunneling calculation is not free of further technical complications, as we now describe: The Euclidean instanton calculation above could possibly describe a number of different processes: *i)* decay of five-dimensional Minkowski space by the spontaneous creation of a SA brane; *ii)* the decay of a SA brane into empty five-dimensional space; or *iii)* a mixing between two distinguished components of the wave-function over five-dimensional geometries, in the sense of quantum gravity. To distinguish between these different possibilities in the saddle-point approximation, one may evaluate the fluctuation determinant around the tunneling solution and determine the number of negative eigenvalues. This computation is complicated in the present situation both by the breakdown of the saddle-point approximation and by the fact that the Lorentzian analysis of the fluctuations around the SA brane indicates the presence of a negative energy ghost. Hence we may expect an infinite number of negative eigenmodes for the bounce solutions. A more careful analysis would be required to determine which of the processes listed above is really relevant. Above, we have simply referred to the process as 'mixing' or 'tunneling' in some general sense. In any event, we expect this quantum tunneling to strongly modify the classical physics, described by the simple solutions discussed, e.g., in section 2. Note, however, that this result may in fact be an indication of the presence of ghosts beyond linear perturbation theory. Indeed, we may follow the converse argument, which simply tells us that since the saddle point approximation breaks down, there should be unstable directions in the phase space of the system. This would then suggest that the strong coupling effects, argued to be important in perturbation theory [17], may not remove the ghost on their own, as long as they leave the self-accelerating solutions in the spectrum.

Another interesting complication arises with the 'wormhole' interpretation of the SA branes mentioned above. Here some further refinement is required in specifying the precise tunneling geometry. Again, the conventional configuration implicitly has the SA brane connecting two disjoint Minkowski spaces. Given two parallel Minkowski spaces, instantons could be constructed in which a SA brane appears in each of the two spaces, but there are two other instantons for which both sides of the SA brane appear in either one of the Minkowski spaces. Hence quantum tunneling seems to naturally lead us to think about configurations where the SA brane connects distant locations within the same five-dimensional space and so makes their role as a 'wormhole' evident. The path integral leading to the probability (4.1) would also include an integral over the position of the instanton. In a conventional setting this would produce a factor of the volume of the five-dimensional space. This result is regulated by dividing out this factor to yield a probability per unit time and per unit spatial volume. However, in the situation where the instanton creates a Lorentzian wormhole, there would be in fact an integral over the position of both sides of the wormhole yielding two volume factors. In this case then, the result still diverges if one asks for the probability that a SA brane appear at given position and time

in the five-dimensional space. Thus the interpretation of these instantons appears less straightforward than usual. Of course, both of these complications are again eliminated if the branes are defined to be $\mathbb{Z}_2$-orbifolds, in which case there is only one side to the branes.

As a consequence of this tunneling processes (irrespective of the sign issues above), we are naturally led to consider a five-dimensional space with a single asymptotic region that contains more than one SA brane (or a single wormhole). At some point in the future evolution of these solutions, the branes will collide with each other. Similarly, SA branes created inside a N brane universe will typically lead to a collision between the two branes, i.e., the self-accelerating brane 'crashing' into the normal brane. Clearly, unless the tunnelling rates found above can be suppressed in some way, this issue remains secondary, since the sheer proliferation of the SA branes, which gobble up space like Witten's 'bubbles of nothing' [45] (even though there may be no 'nothing' here when we impose $\mathbb{Z}_2$ symmetry), will end up completely destroying the bulk environment.

Above we considered tunneling processes in which a SA brane is nucleated in empty five-dimensional Minkowski space. This process would allow the quantum creation of such a brane in any region of Minkowki space. In particular, consider the N-branch where the brane encloses a region of Minkowski space with finite spatial extent, as described in section 2. The minimum radius of this region (at $t = 0$) is H_-^{-1}, which also corresponds to the size of the corresponding Euclidean solution. Similarly the minimum radius of the SA brane (or maximum radius for the Euclidean bounce solution described above) is $H_+^{-1} < H_-^{-1}$. Hence it is easy to fit a SA brane worldvolume inside that of a N brane, for a fixed value of the tension, σ. Given the latter fact, it is straightforward to extend the above calculations for the spontaneous appearance of SA branes in the bulk space enclosed by a N brane. In fact, ΔS is precisely the same as given in (4.9) and so the conclusions are the same for this case.

To close this section, we list some more exotic possibilities for tunneling processes. First off, for $\mu < 0$ there is a very interesting possibility for mixing between unregulated bulks with naked singularities and their regulated variants involving SA branes. The brane trajectories will come from a Wick-rotated version of (3.11) and will have a smooth intrinsic geometry, even though they close off at the singularity. However, near the singularity their extrinsic curvature must diverge. If we were to ignore the action on the singularity we would expect to find a finite negative action. Clearly, this answer would be sensitive to higher order corrections and hence unreliable. Yet, we would expect in general that solutions with large negative actions should exist, again yielding strong mixing, and allowing for a formation of a naked singularity. Note, that the usual positive energy theorems [32], which normally enforce cosmic censorship, do not apply in the DGP model.

For the case $\mu > 0$ we would expect that nonperturbative processes describing bulk black hole formation are possible on the N branch. These however may be suppressed by the thermodynamic factors which do apply on the normal branch, in the bulk. Another possibility would involve transitions between collapsing solutions that would bounce to those which crunch, and back. Without a precise calculation of the solutions it is not clear what the significance of this instanton is. Indeed, for sufficiently small $\mu > 0$ there are no crunching, confined trajectories to begin with.

5. Discussion

The DGP model originally captured the attention of theoretical physicists by offering an explanation of the cosmic acceleration that was distinct from the standard one with a cosmological constant. An important element of the model was seen to be its internal consistency. However, recently flaws in this consistency have emerged. A detailed analysis of the perturbative fluctuations revealed that the spectrum of the self-accelerating solutions contain ghost-like and tachyonic excitations [14]. In the present paper, we have taken a new perspective on the model and rather than focus on the four-dimensional physics of the branes, we have studied the theory from a five-dimensional point of view. With this new perspective, simple calculations straightforwardly led us to see a number of new pathologies of the DGP model.

One simple observation in section 2 was that the conventional SA brane connecting two asymptotically flat regions can be regarded a 'wormhole'. As such, DGP gravity faces certain problems which are inherent to such wormhole configurations [19]. One issue that is often discussed in this context is the necessity for negative energy densities to support a wormhole. As was also discussed in section 2, the effective stress tensor (2.8) of the DGP branes does not satisfy any positive energy conditions and in fact, the SA brane behaves as a brane with a negative effective tension. Another issue that arises with wormholes is the possible appearance of closed time-like curves (CTC's). Here one must note that the present wormholes differ from those conventionally discussed [19] since the size of the wormhole throat is not fixed, i.e., the brane follows a hyperbolic trajectory in Minkowski space. However, it is straightforward to show that this dynamic behaviour does not prevent the appearance of CTC's. In section 4, we also found that the wormhole geometries lead to additional divergences in the semiclassical tunneling calculations. Of course, all of these problems are eliminated if the DGP branes are $\mathbb{Z}_2$-orbifolds, in which case, the brane is actually a boundary of the five-dimensional spacetime.

In section 3, we provided a construction of SA solutions where the five-dimensional energy was arbitrarily negative. While these solutions are singular at $a = 0$ for a sufficiently large and negative energy, they still admit well-behaved smooth initial data and so there do not appear to be any obvious reasons not to include these solutions. The appearance of a singularity is simply a shortcoming of the model and a reminder that we are working with an low energy effective theory. Hence our conclusion is that regarded as a five-dimensional theory of gravity and branes, the spectrum of the DGP model is unbounded from below. This reinforces the expectation developed from the fluctuation analysis [14] that the SA brane should be unstable to decay by both classical and quantum processes. In particular, we have found explicitly (see the appendix) that the negative mass configurations can be smoothly attained by classical radion variation from SA branes. Hence one can regard the solutions in section 3 as nonperturbative excitation of the normalizable radion mode, going beyond the original linearized analysis and extending the associated instability to the fully nonlinear regime of the theory.

As our construction in section 3 shows the addition of DGP branes to the five-dimensional Einstein gravity has allowed the theory to resolve the singularity of negative

mass Schwarzschild solution. From the point of view of a quantum gravity, this was long ago argued to be a disaster for the theory [52], for essentially the same reasons as elaborated above. Namely, the complete theory would admit negative energy states and the empty space would no longer be a stable vacuum. One might wonder if there might be some super-selection rules that keep the energy positive. After all, for desired phenomenological application, one needs SA branes which are really big, even with negative energy, and so might be hard to produce. But the tunneling calculation, and classical radion evolution seem to support that the opposite is true, and that the system will probe arbitraily low energies.

While the explicit constructions of section 3 are very symmetric, it is clear that they only represent a small corner of a vast space of negative energy solutions. The key point is that in the approach elucidated in section 2, the SA brane excises a finite region at the center of the five-dimensional bulk and so one can allow any manner of singularities there. A simple extension of our symmetric solutions would be to choose the bulk geometry as a negative mass Schwarzschild but to position the singularity away from the center of the space and closer to the minimum radius reached by the brane. Determining the precise trajectory of the SA brane would be computationally a more complicated task but it seems clear that the result will be a negative energy solution where, rather than being uniformly spread, the excitation is localized in one region of the brane. Ergo, inhomogeneities with arbitrarily negative energies should also form. Based on this and the connection to the perturbative analysis, one should expect that to encode such modes there should be non-linear solutions which represent excitations of the higher unstable modes as well, i.e., ghost or tachyon modes with nontrivial angular momentum on the three-sphere.

It is interesting to contrast our present results on negative energy states with previous observations of effective negative energies associated with matter configurations on SA branes [53, 54]. In both cases, these nonlinear solutions showed behaviour corresponding to a negative energy density at some intermediate scale. However, this effect is not indicative of the overall mass or energy measured at asymptotic distance scales. In the case of the domain walls [54] — see also [24] – where the full five-dimensional solution is known, one clearly sees that the $5D$ wormhole geometry of the SA brane screens the effective negative energy. That is, since the asymptotic geometry is simply $5D$ Minkowski space, from the five-dimensional perspective advocated here, we see that these configurations correspond to precisely zero energy states. It is still interesting to ask if these effective negative energies appearing at intermediate scales can produce instabilities through some local processes on the brane. However, we would argue that this is unlikely to be the case. By considering domain walls connecting regions with different brane tensions, it is clear that the negative energy is screened on the scale set by the size of the bubble surrounded by the domain wall.[6] Hence it seems that this screening will prevent the formation of such bubbles in dynamical processes.

In section 3, we also saw how a *positive* energy bulk would lead to problems of a very

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[6]The solutions given in [54] describe bubbles of the size of the cosmological horizon. By considering the analogous constructions with domain walls separating regions with different brane tensions, we find solutions where the size of the bubble is disentangled from the horizon scale.

different nature. Again, as shown in the appendix, one can easily generate a positive mass in the bulk by exciting the radion on the SA background. Typically, the cosmological solutions we found run into a pressure singularity on the brane, at some finite value of the scale factor. This represents an absolutely disastrous instability on the SA branes, as can be seen by considering the evolution of a scalar field on the brane. As the pressure diverges, so the scalar field starts to behave as if it has a diverging tachyonic mass on all scales! The pressure singularity can also appear for a positive energy bulk on the N branch, but in that case the scalar field picks up a diverging positive mass, and simply decouples from the dynamics of the universe.

Yet another pathology came from standard semi-classical calculations showing that there should be a rapid spontaneous creation of SA branes in any region of five-dimensional Minkowski space. Of course, as discussed in the previous section, these calculations are not free of various ambiguities inherent in the Euclidean path integral in quantum gravity. However, having alerted the reader to these caveats, we continue to consider the implications of the results. Note that these explicit calculations are restricted to the zero-energy sector of the five-dimensional theory. Hence this instability is complementary to the instabilities which one would expect to arise from negative energies discussed above. In any event, these results suggest that empty five-dimensional Minkowski space is not the vacuum and not even close to the vacuum of the DGP model as a five-dimensional theory of gravity. Of course, this is a disturbing result that calls into question any intuition or results derived from classical solutions in the DGP theory. In particular, it casts serious doubt on the SA solutions, which are really the reason behind these pathologies. Interestingly, this is reminiscent of the situation encountered in higher derivative gravities in $4D$, where classical de Sitter solutions supported by higher derivative terms were also linked to vacuum instabilities [55]. There, a cure which was devised was to exclude such configurations from the allowed set of solutions of the theory.

Hence what are we to conclude from these results? In fact, the only firm conclusion we wish to draw is that as a theory of five-dimensional gravity the DGP model is not consistent, as it displays some severe pathologies. We must remind ourselves, however, that this model is only a low-energy effective model and not a complete theory of gravity or other physics. As we noted above, we are readily reminded of this status by the appearance of spacetime singularities from rather generic and uneventful initial data. Could it be then that the DGP model has a UV completion, which evades all of the problems which we have elucidated here? While this may seem a rather unlikely eventuality, let us point out a few simple examples where in fact this might be the case.

The first example is provided by [56], which was motivated by the appearance of negative-tension branes in the original Randall-Sundrum model [61]. Of course, the latter have some obvious instabilities that were eliminated by invoking a $\mathbb{Z}_2$-orbifold boundary condition. The authors of [56] found a new dynamical instability, which applied even with this boundary condition, by studying black holes falling in on negative tension branes from the bulk. This process was found to produce a catastrophic 'big crunch' singularity which caused the entire bulk space to collapse. However, they also observed that, if the negative tension brane in the RS model is actually realized by an orientifold of a higher dimensional

geometry in a string theory construction, then this instability should not be expected to arise. That is, this pathology of the RS model is only a pathology of the low energy description and so would not play a role in a string theory realization of the same physics.

As a second example, we address directly one of our DGP pathologies, albeit not with a UV completion of the model. Imagine extending (2.1) by adding the following Gauss-Bonnet term[7] to the brane action:

$$\frac{\beta}{128\pi^2} \int_{\text{brane}} \sqrt{-\gamma} \left(\mathcal{R}_{\mu\nu\rho\sigma} \mathcal{R}^{\mu\nu\rho\sigma} - 4\mathcal{R}_{\mu\nu} \mathcal{R}^{\mu\nu} + \mathcal{R}^2 \right) \tag{5.1}$$

where β is a dimensionless constant. In fact, the effect below could be accomplished by adding any term involving squares of the intrinsic curvature. A generic term would, however, modify the behaviour of gravity on the brane at short (and long) distances and also lead to new ghost excitations in the UV. Both of these complications could be evaded with the Gauss-Bonnet term (5.1), which because of its topological nature does not effect the dynamics. However, if this term is carried through the calculations of section 4, the final result would be that ΔS is shifted by a constant, i.e., (4.9) is replaced by

$$\Delta S' = -\frac{8\pi^2}{9H_+^4} \left(12 M_4^2 H_+^2 + \sigma \right) + \beta \,. \tag{5.2}$$

Since the standard contribution is determined by the microscopic parameters, M_5, M_4 and σ, once these are fixed to produce a certain dynamics, we have the freedom to tune β to a sufficiently large positive value so that $\Delta S' > 0$. As a result, the rapid spontaneous creation of SA branes would be suppressed and so this pathology would be removed. While it may seem unnatural that β should have to be tuned in this way rather than simply being $O(1)$ (or perhaps $O(128\pi^2)$ given our normalization), we would point out that the DGP model already required a certain amount of fine-tuning, e.g., in the ratio M_5/M_4 to produce a phenomenologically viable model. So while we have not addressed the question of a UV completion, we have alluded to a theoretical mechanism which may suppress[8] the tunneling processes[9] considered in section 4. We should also mention, that in principle one might also seek to suppress the nucleation of SA branes, or exclude this channel altogether, in a similar vain as the Witten's bubbles of nothing are suppressed in the usual KK compactification. This can be done by adding other bulk degrees of freedom, for example fermions, that will twist nontrivially in a bulk with an SA brane, by seeking boundary conditions sensitive to the extrinsic curvature. Such terms could, at least in principle, change the action of the Euclidean SA configuration. It would be interesting to explore this in more detail.

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[7]Gauss-Bonnet terms in the bulk have been widely explored in various braneworls setups [57]. Here we only add them to the brane action, and not in the bulk. Thus they are purely topological, being relevant only for the quantum dynamics of the theory.

[8]Recall that any tunneling at all gives rise to configurations where SA branes will collide.

[9]Having pointed to a positive influence of a Gauss-Bonnet term, we should also reiterate the concern about the theory where the regularity of the solutions found from analyzing leading order terms in the derivative expansion is so sensitive to the inclusion of higher-dimension operators, on the brane or in the bulk. This, in the very least, underlies the importance of designing a UV completion before jumping to phenomenological applications of the solutions of the theory at the present stage.

Our discussion then brings to the fore the importance of finding a proper UV completion for the DGP model. While there are claims that DGP gravity can be realized in certain string theory constructions [58, 59] — see also [27] — more recent analyses yielded the argument that in fact it would be impossible to find a UV completion for this model [29].[10] Clearly it is very important to resolve this issue and in fact, at present, this seems to be the most pressing question to be addressed with respect to DGP gravity. We contrast this situation with that of, e.g., Randall-Sundrum 'gravity' [61]. The RS model is known to suffer from gravitational instabilities involving unstabilized negative tension branes. However, the basic five-dimensional model still serves as a useful testing ground for new ideas in particle phenomenology (for reviews see [62]), which stay away from the complications of gravitational nonlinearities. Moreover, with the development of the stabilizing mechanisms [64], the instabilities can be put under control at low energies. Further in the RS case, we can rest assured that string theory provides consistent constructions [63] which produce very similar physics to the simple five-dimensional models and which evade the problems found in the low energy effective theory. On the other hand, the DGP model was constructed precisely as a testing ground for new gravitational physics and although our discussions address nonlinear instabilities, we know that these instabilities link up with the ghost problems found in the linear theory [14]. Hence one must object that in its present form the DGP model does not provide a consistent framework to test new gravitational ideas, as long as it permits 'unchecked' SA branes. Thus we re-iterate that the most pressing question to resolve about DGP gravity is whether a UV completion exists, and, if so, what are the UV characteristics of the theory. For example, perhaps a sensible UV completion can guide us to make some simple modifications of the original DGP model which will ameliorate the problems discussed above. In any event, the present discussion reinforces that it is premature to use the DGP model to develop any detailed cosmological tests.

Going beyond the DGP model, we note that the perspective and analysis that we applied here can be utilized broadly in many other braneworld extensions of four-dimensional Einstein gravity. Examples which combine DGP and RS features were studied in, e.g., [5, 20]. Our approach would give a quick diagnostic of the consistency of such models. One expects that in the realizations which contain perturbative ghosts and SA branches, similar negative energy problems and singular evolutions will arise. It would be interesting to check this explicitly. Also, it would be interesting to explore higher-codimension models with brane localized terms. Such models have been recently investigated in [65], where a construction without ghosts was provided. Yet it appears that for some boundary conditions on the brane a nonlinear analysis as in section 3 may yield negative energy solutions. Finding out the precise link of these configurations could shed more light on the interconnectednes of perturbative ghosts and possible instabilities. Ultimately, learning more about such problems may give us an opportunity to test the robustness of General Relativity. While seeking modified gravities is an interesting endavour which may be motivated by various cosmological and phenomenological problems, it's also a dangerous

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[10]The applicability of this analysis to DGP gravity was subsequently called into question in [60].

venture. Without a reasonable UV completion to guide us, many of these models will simply provide pathological theories that do not have well behaved low energy limits, and remain altogether unreliable. Yet, at this time we still do not know for a fact if the successes of General Relativity really require General Relativity, or might be reproduced by a more exotic structure. Searching for more clues to point us either way thus remains an interesting effort.

A. The radion and the Schwarzschild bulk

In section 3 we examined in some detail the behavior of a brane embedded in a bulk Schwarzschild geometry. For very small values of the black hole mass, we observed that the bulk geometry is only slightly deformed from flat space[11] (2.4) and the brane trajectory is only slightly perturbed from the standard cosmology (2.6). Hence in this regime, we should be able to connect the new solutions of section 3 with the fluctuation analysis of [14, 33]. We commented above and will demonstrate here with a detailed calculation that the small μ perturbation corresponds to the homogeneous mode of the radion, as studied in [14, 33]. Further, we observe that on the self-accelerating branch, these perturbations are normalizable, whereas on the normal branch they are not. Perhaps this is not surprising since the normalizable radion decouples on the normal branch, but not on the SA branch.

Before embarking on the detailed calculation, let us note that the radion field roughly measures fluctuations in the brane's position in the bulk. In going from a Minkowski bulk to a Schwarzschild bulk, we would expect the brane to respond by being drawn towards the black hole, and so clearly the radion field will be excited first and foremost. Secondly, on the self accelerating branch the black hole has itself been cut out, only leaving its long range fields in the asymptotic region. This means that the boundary conditions 'deep inside the bulk' correspond to boundary conditions in asymptotically flat space, i.e., they

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[11]As noted below, this only really applies on the SA branch.

are identical to those for a five-dimensional Minkowski bulk geometry. Therefore, the Schwarzschild solution must correspond to a *normalizable* perturbation. Of course, this is not so on the normal branch, which retains the interior region. Hence in that case the boundary conditions deep inside the bulk differ: for the Schwarzschild bulk, we have a black hole while with a Minkowski bulk, we simply have empty flat space. Therefore, the Schwarzschild solution on the normal branch must correspond to a *non-normalizable* perturbation, which simply means that this mode enters the nonlinear regime deep in the bulk.

We will now elaborate our claims with an explicit calculation. When the mass parameter, μ, is small, the bulk solution (3.1), and the brane equation of motion (3.9) clearly correspond to vacuum perturbations about the background solutions with a Minkowski bulk ($\mu = 0$). Linearizing in μ, the bulk solution (3.1) becomes

$$ds^2 = d\bar{s}^2 + \frac{\mu}{r^2}(dt^2 + dr^2) + \mathcal{O}(\mu^2) \tag{A.1}$$

where $d\bar{s}^2$ is the background Minkowski metric (2.4), written in global coordinates. We now change to a de Sitter slicing of Minkowski by introducing new coordinates as follows

$$r = \frac{e^{\epsilon Hy}}{H}\cosh(H\tau), \qquad t = \frac{e^{\epsilon Hy}}{H}\sinh(H\tau). \tag{A.2}$$

The background bulk metric now takes the form given in [14, 33]

$$d\bar{s}^2 = e^{2\epsilon Hy}\left[dy^2 + \bar{\gamma}_{\mu\nu}dx^\mu dx^\nu\right], \tag{A.3}$$

where $\bar{\gamma}_{\mu\nu}$ is a spherical slicing of $4D$ de Sitter (see equations (2.5) and (2.6)), and $\epsilon = \pm 1$ depending on whether we are on the N branch ($\epsilon = -1$) or the SA branch ($\epsilon = +1$).

Using (A.1), the Schwarzschild geometry now corresponds to a perturbation of the form

$$\delta g_{yy} = \delta g_{\tau\tau} = \mu H^2(1 + \tanh^2(H\tau)), \qquad \delta g_{y\tau} = 2\mu H^2\tanh(H\tau), \tag{A.4}$$

and $\delta g_{ab} = 0$ otherwise. In [14, 33], we predominantly worked in Gaussian-Normal (GN) gauge, for which, $\delta g_{ay} = 0$. Since we wish to identify the Schwarzschild bulk with vacuum perturbations appearing in [14, 33], it is therefore convenient to transform to GN gauge as follows

$$y \to y + \eta, \qquad x^\mu \to x^\mu + \xi^\mu \tag{A.5}$$

where

$$\eta = -\frac{\mu}{2}\epsilon H e^{-2\epsilon Hy}\left[1 + \tanh^2(H\tau)\right] \tag{A.6}$$

$$\xi^\mu = \frac{\mu}{4}e^{-2\epsilon Hy}D^\mu\left[\tanh^2(H\tau) + 4\ln(\cosh(H\tau))\right] \tag{A.7}$$

Here D_μ is the covariant derivative for the $4D$ de Sitter metric, $\bar{\gamma}_{\mu\nu}$. The metric perturbation now takes the form given in [14, 33]

$$\delta g_{ay} = 0, \qquad \delta g_{\mu\nu} = e^{\epsilon Hy/2}h_{\mu\nu}(x, y) \tag{A.8}$$

where

$$h_{\mu\nu}(x,y) = \left[D_\mu D_\nu + H^2 \bar{\gamma}_{\mu\nu} \right] \phi(x,y), \qquad \phi(x,y) = e^{-\epsilon Hy/2} \hat{\phi}(\tau), \tag{A.9}$$

$$\hat{\phi}(\tau) = -\mu H \sinh(H\tau) \int^\tau \frac{d\lambda}{\cosh^3(H\lambda)\sinh^2(H\lambda)} \tag{A.10}$$

It is easy to check that $(D^2 + 4H^2)\hat{\phi}(\tau) = 0$, which means that $h_{\mu\nu}(x,y)$ is transverse-trace-free. Indeed, if we compare this with the perturbations given in [14, 33], we see that the mode $\phi(x,y)$ can indeed be identified as the radion. On the SA-branch ($\epsilon = +1$), the mode decays for large y, and is therefore normalizable. In contrast, on the N branch ($\epsilon = -1$), the mode grows for large y and is not normalizable!

It may come as a surprise that both the radion field (A.10) and the energy of the solution (3.3) are linear in the Schwarzschild mass parameter μ. Examining the effective lagrangian (3.71) of [14], one sees that it is quadratic in the radion, to leading order. Hence the naive expectation would be that the Hamiltonian is also quadratic, as opposed to linear, in μ. Without going into any great detail, the source of this 'discrepancy' is the fact that the background solution is time-dependent. That is, even though the bulk spacetime (2.4) has a Killing time,[12] the embedding of the standard SA brane does not respect this symmetry. Given this time dependence, in fact, we should in general expect that the Hamiltonian is linear in perturbations about the background. This is most easily illustrated with the simple example of a classical mechanics problem with $\mathcal{H} = p\dot{q} - \mathcal{L}$. If we perturb about a specific solution (p_0, q_0), it is a straightforward calculation to show that in general $\delta\mathcal{H} = \dot{q}_0\,\delta p - \dot{p}_0\,\delta q$. Hence unless the background solution is static, we should expect the shift in the energy to be linear in the perturbations. Of course, the same analysis should apply directly to the present problem with perturbations around the standard cosmology of SA brane (with $\mu = 0$).

In summary we have confirmed our naive expectations: namely that a Schwarzschild bulk corresponds to a normalisable radion on the SA branch, and a *non*-normalizable radion on the N branch.

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[12]Of course, this is also the asymptotic time conjugate to the energy which we measure with μ.

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